Mikhail Alekseevich
LAVRENT' EV

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On his sixtieth birthday

Mikhail Alekseevich Lavrent'ev was born on November 19, 1900 in Kazan, into the family of a teacher of mathematics. Under the influence of his father, Alexei Lavrent'ev, he became interested from his earliest years in mathematics, mechanics and astronomy.

Lavrent'ev completed six years at the Kazan commercial college and then in 1918 entered the physics-mathematics faculty of the University of Kazan.

At the University of Kazan, Lavrent'ev was most influenced by the professors of mathematics and mechanics, N.N. Parfent'ev, D.N. Zeiliger, and E.A. Bolotov.

In 1921, Lavrent'ev transferred to the mathematics department of Moscow State University. Working at that time at Moscow State University was Nikolai Nikolaevich Luzin, famous for his investigations into the theory of functions of a real variable. The first scientific interests of the young scientist were built up under the influence of Luzin.

Lavrent'ev was also greatly influenced by S.A. Chaplygin, who at that time was directing the instruction of mechanics at Moscow University.

Upon graduating from Moscow University in 1922, Lavrent'ev worked at the Higher Technical Institute of Moscow, first as an assistant, and later as docent of the chair of mathematical analysis.

At the same time, during the period 1922 to 1926, Lavrent'ev was an aspirant (doctoral candidate) under Luzin, who held the chair of analysis in the theory of functions of a real variable. At the beginning of 1926 he defended his candidate's thesis, in which questions in the theory of geomorphism were developed.

With the successful completion of his dissertation, Lavrent'ev was given a scientific assignment to France, in February 1927. Here he spent half a year. He met the famous French mathematicians, Danjois and Montel, attended the lectures of Goursat, Borel, Julia, and others, and took part in the seminar on the theory of functions which was directed by Hadamard.

At the end of 1927 Lavrent'ev was elected privat-docent of the chair of analysis of the Moscow State University, and also a member of the

Mathematical Society. At Moscow University Levrent'ev gave his first course in the theory of conformal transformations.

In 1928 Lavrent'ev took part in the International Mathematical Congress at Bologna (Italy), where he presented a scientific paper on his investigations in the area of conformal transformations.

In the fall of 1929 Lavrent'ev received the chair and title of professor in the D. I. Mendelev Chemical-Technological Institute. Here, over a period of two years, he gave various courses in mathematics.

At the end of the twenties, our country began a great effort to build up Soviet aviation. Much scientific effort was attracted to the work of the Joukowski Central Aero-Hydrodynamic Institute. The director of the theoretical division of this institute, Chaplygin, invited Lavrent'ev to become senior engineer of the division. The work of this division had a great influence, in its time, on the development of aviation, shipbuilding and other sciences of our nation.

Here Lavrent'ev worked in collaboration with the talented young scientists, M. V. Keldish, N. E. Kochin, L. I. Sedov, and others. Independently, and together with them, Lavrent'ev worked on the theory of flow over wing sections, investigations into impact on water, applicable to the landing of seaplanes, and other problems.

In 1932 Lavrent'ev was awarded the degree of Doctor of Technical Sciences, and in 1933 the degree of Doctor of Physical-Mathematical Sciences. In those years he headed the department of the theory of functions at the Mathematical Institute of Moscow University. He worked at Moscow University from 1931 to 1939 and from 1950 to 1958, and for some time directed the unified department of analysis.

From 1933 Mikhail Alekseevich worked at the Steklov Mathematical Institute of the Academy of Sciences of the USSR, directing the department of the theory of functions.

In 1939, at the invitation of A. A. Bogomolets, president of the Academy of Sciences of the Ukrainian S. S. R., Lavrent'ev came to Kiev; he was elected a full member of the Academy of Sciences of the Ukrainian S. S. R. and made director of the Institute of Mathematics of the Academy.

During the war years, the Institute of Mathematics together with other establishments of the Academy of Sciences of the Ukrainian S. S. R. was evacuated to Ufa, the capital of the Bashkir A. S. S. R. Here Lavrent'ev worked hard solving various problems connected with the defense effort, and on problems connected with the perfection of artillery weapons.

After the return of the Academy of Sciences of the Ukrainian S. S. R.

to Kiev in 1944, M.A. Lavrent'ev directed the work of the Institute of Mathematics. The experimental laboratory of the Institute of Mathematics, built on his initiative, quickly became a center of many investigations. In this laboratory, students of higher educational institutions of Kiev, Leningrad, Dniepropetrovsk and other cities obtained practical training. Mathematicians, physicists, specialists in mechanics, in strength of materials and construction, as well as biologists, all have worked in the laboratory.

In 1945 Lavrent'ev was elected vice-president of the Academy of Sciences of the Ukrainian S. S. R. As vice-president, he exerted great efforts for the development of science in the Ukraine. During that time, Lavrent'ev was carrying on a number of researches, also directing the work of the Institute of Mathematics of the Academy of Sciences of the Ukrainian S. S. R., and carrying on teaching work at the Kiev State University. Seeing clearly the role of computing technology in the progress of science and technology, Lavrent'ev paid much attention to the development of computing machines in the U.S.S.R. On his initiative, there began in Kiev the construction of the first Soviet small electronic computing machine, which was built in the Electrotechnical Institute of the Academy of Sciences of the Ukrainian S. S. R., under the direction of S.A. Lebedev.

The scientific activity of Lavrent'ev in the Academy of Sciences of the Ukrainian S. S. R. had a significant influence on the development of mathematics and mechanics in the Ukraine.

For his outstanding work in the field of mathematics and mechanics, Lavrent'ev was elected a full member of the Academy of Sciences of the U.S.S.R. in 1946.

In 1949 Lavrent'ev returned to Moscow and was appointed director of the Institute of Exact Mechanics and Computational Technology of the Academy of Sciences of the U.S.S.R. Here, on his initiative, work was begun by Soviet mathematicians on problems of programming, and also the work on the development of domestic digital machines was extended.

From 1951 to 1956 Mikhail Alekseevich was twice elected Academician-secretary of the Division of Physical-Mathematical Sciences of the Academy of Sciences of the U.S.S.R.

Following the resolution of the Twentieth Congress of the Communist Party on the development of industry and utilization of resources of Siberia and the Far East, Lavrent'ev, together with Academicians S.L. Sobolev and S.I. Khristianovich, proposed the creation of a Siberian division of the Academy of Sciences of the U.S.S.R.

By a decision of the Council of Ministers of the U.S.S.R., Lavrent'ev was appointed in 1957 to be chairman of the organizing committee and director of construction of the science center of the Siberian division of the Academy of Sciences of the U.S.S.R. At the same time he was installed as director of the Institute of Hydrodynamics of that division. In September 1957, Mikhail Alekseevich was elected vice-president of the Academy of Sciences of the U.S.S.R.; in May 1958 he was elected Chairman of the Presidium of the Siberian division of the Academy of Sciences of the U.S.S.R.

In March 1958 Lavrent'ev was elected a deputy of the Supreme Soviet of the U.S.S.R.

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Lavrent'ev's first scientific results were obtained in the theory of functions of a real variable; in 1924 he published an article on the convergence of transfinite series, and also an investigation on the descriptive theory of functions. In 1925 Lavrent'ev published a theorem, now bearing his name, on the sub-classes in Baire's theory of numbers. This result was based on the investigation of the geomorphism of various classes of numbers.

In the same year he constructed an example of an ordinary differential equation which has a solution at every point of the plane and in addition is not singular in any neighborhood of it.

However, Lavrent'ev's scientific interests soon changed to the theory of functions of a complex variable, in which he set a number of scientific directions. In one of his first papers Lavrent'ev, using a theorem of Luzin and I.I. Privalov, established the geometrical conditions for the invariance of boundary manifolds of zero measure in conformal transformations, and gave a quantitative determination of the deformation of the measure in these transformations.

To Lavrent'ev, together with Keldish, is due the example of the construction of a region of arbitrarily small diameter bounded by a curve such that, in a normalized transformation of that region onto the unit circle, every arc of the bounding curve transforms into an arc of equal length on the circle.

The development of papers on the properties of boundary points in conformal transformations resulted from Lavrent'ev's investigations into the variational principles of these transformations; he gave a number of variational principles which are important in both the theoretical and applied sense.

With the help of variational principles, Lavrent'ev continued his investigations into the properties of boundaries in conformal transformations. Thus, using the generalized principle of Lindelof, he obtained an estimate of the derivatives of a mapping function on the boundaries of a region. The variational-geometric methods of Lavrent'ev were applied by him in extremal problems of the theory of functions.

Lavrent'ev's investigations into the theory of approximations in the field of functions of a complex variable laid the foundations of a new line of work; of special significance here is his work on convergent series of polynomials.

Significant influence on the further development of mathematics was exerted by Lavrent'ev's investigations into the theory of quasi-conformal transformations, i.e. transformations in which a small circle transforms into a small ellipse with given eccentricity or with other given parameters.

Lavrent'ev extended his methods from the theory of conformal transformations to the theory of quasi-conformal transformations. He established for them the same variational principles, proved the same fundamental theorems on the existence of transformations of given regions onto a circle or strip, and investigated the properties of the boundaries. This theory made it possible to obtain new results in the theory of analytic functions, in the theory of conformal transformations of Riemann surfaces.

In the theory of three-dimensional quasi-conformal transformations, described by strongly elliptic systems of partial differential equations, Lavrent'ev established variational principles as in the plane case. This enabled him to determine certain solutions of Dirichlet's problem in three-dimensional space.

Finally, mention should be made of the paper, with Keldish, "On the stability of solutions of the Dirichlet problem with respect to a change in the boundary", in which there was obtained the solution to a difficult and very interesting problem, important in mathematical physics.

Lavrent'ev's first investigations in mechanics were in connection with the construction of aerodynamic profiles. In 1933 he published the first paper, in which the construction of the flow over an arc of given shape is reduced to an integral equation; a converging sequence of computations was very nicely adapted to its solution.

A second paper by Lavrent'ev in 1934 was concerned with the variational problem of determining the optimum profile shape. He also investigated the problem of the flow over a system of two arcs.

The theory of the impact of a rigid body on the surface of an incompressible fluid was developed by Lavrent'ev, together with Keldish and Sedov, in 1935-36. These investigations were connected with the landing of airplanes on water. The distribution of velocities and impulses over a plate representing a pontoon was determined.

The work of Lavrent'ev and Keldish in the theory of an oscillating wing was a development of the investigations of Chaplygin and Prandtl on the non-stationary motion of a wing. In this work the theory was made more precise with the help of the method of conformal transformations. It was shown that for a periodically changing angle of attack of the wing, the mean value of the lift over one period is equal to that value which is obtained by putting the mean angle of attack into Joukowski's formula. In addition, it was shown that there exist regimes of oscillation for which a horizontal force (drag) arises.

In the paper by Keldish and Lavrent'ev, "On the motion of a wing below the surface of a liquid", a thin wing is in motion near the surface of an incompressible fluid on which small waves are formed; the angle of attack is also taken to be small; to determine the flow around the wing it is replaced by a set of vortices. Under these assumptions, the characteristic stream function is represented in the form of an integral of the stream functions of isolated vortices and their density distribution. The lifting force and the drag of the wing are determined; detailed calculations are carried out for a thin wing in the form of a circular arc. It is also shown that for large values of the Froude number the wave drag of the wing tends to a finite limit.

To Lavrent'ev is due a fundamental result in the nonlinear theory of waves on the surface of a liquid. In 1944 he proved the theorem of the existence of a solitary wave. This investigation terminated a discussion which had gone on over a span of a hundred years following the initial investigations of Scott-Russell. The name of Lavrent'ev is closely connected with an important area in the theory of fluid motion with free boundaries. In these investigations, there is introduced an approximation for the square of the velocity by a function which involves the form of the boundary and its derivatives. This idea of Lavrent'ev is connected with his investigations into approximations of conformal transformations of narrow strips.

With the growth of Soviet science and modern technology, the area of scientific interests of Lavrent'ev also increased.

A series of investigations by Lavrent'ev, published in the period from 1936 to 1957, was related to the study of dynamic phenomena connected with very high velocities, in particular, the study of explosions

and the cumulative effects of explosions. Lavrent'ev noted that viscosity and plasticity became unimportant in the case of motion at very high velocities and pressures. Therefore, in many cases it can be assumed that the medium behaves like an incompressible fluid. In this approach, the dynamics of explosion phenomena can, in a certain region, be investigated by the methods of classical hydrodynamics. This idea proved fruitful for many problems. Thus, it allowed Lavrent'ev to explain the cumulative effect of an explosion.

Making use of theoretical considerations and also the results of experiments, Lavrent'ev determined the effect of a pulsating jet on an obstacle, and showed that the resistance to penetration is determined by the inertial properties of the medium. This approach made it possible, for the first time, to obtain the relations for computing the directional effects of exploding charges. In the case of pulsating jets, the velocities are of the order of 10 km/sec. In a paper in 1959, Lavrent'ev investigated impacts at velocities of the order of 50 to 100 km/sec; this is the problem of the penetration of a cosmic spaceship by small meteorites. The distribution of deformation velocities in the body receiving the impact is determined, and so is the impulse which it receives.

Examining the results of an underwater explosion, Lavrent'ev pointed out the singular form of instability of the round metal pipe which was used as the charge container. The pipe became unstable with the formation of corrugations on the periphery, whose number increased in proportion to the proximity to the point of attachment of the charge. The investigations carried out by Lavrent'ev together with A. Iu. Ishlinski showed that in the case of a suddenly applied stress there is an instability of the pipe, corresponding to high harmonics. The form of the instability determined was confirmed by experiments.

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The scientific, organizational and pedagogical services of Lavrent'ev have received high public appreciation. The Soviet government decorated him with three Orders of Lenin, four Orders of the Red Banner of Labor, and the Order of the War of the Fatherland, Second Degree.

The personal qualities of Mikhail Alekseevich gained him the deep respect and love of his comrades in work, his students, and all those who had occasion to come to him on scientific or public questions.

The scientific community of our country sincerely congratulates Mikhail Alekseevich on his sixtieth birthday, and wishes him good health and continued success in his work.

List of the Scientific Works of M.A. Lavrent'ev*

1924

Sur la recherche des ensembles homéomorphes. *C.R. Acad. Sci., Paris* Vol. 178, pp. 187-190.

Sur la représentation des fonctions mesurables B par les séries transfines de polynomes. *Fundam. Math.* Vol. 5, pp. 123-129.

Contribution à la théorie des ensembles homéomorphes. *Fundam. Math.* Vol. 6, pp. 149-160.

1925

Sur les sous-classes de la classification de M. Baire. *C.R. Acad. Sci., Paris* Vol. 180, pp. 111-114.

Sur une équation différentielle du premier ordre. *Math. Zs.* Vol. 23, pp. 197-209.

1927

Sur la représentation conforme. *C.R. Acad. Sci., Paris* Vol. 184, pp. 1407-1409.

Sur un problème de M.P. Montel. *Ibid.* pp. 1634-1635.

Sur quelques problèmes du calcul des variations. *Ann. di Mat.* Series 4, Vol. 4, pp. 7-28.

1928

Obshchii ocherk razvitiia teorii funktsii kompleksnogo peremennogo v SSSR za vremya s 1917-1927 gg. (General outline of the development of a complex variable in the USSR in the period from 1917 to 1927). *Mat. Sb.* Vol. 35, supplementary number, pp. 5-20. (In collaboration with I.I. Privalov).

Uspekhi teorii funktsii deistvitel'nogo peremennogo v SSSR (Advances in the theory of functions of a real variable in the USSR). *Ibid.* pp. 21-42. Bibliography, pp. 40-42. (In collaboration with D. Menshov).

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* The present list is a portion of the full bibliography of the papers of M.A. Lavrent'ev, prepared by A.P. Epifanova and published in the journal, "Prikladnaya mekhanika i tekhnichiskaya fizika" (Applied Mechanics and Technical Physics), No. 3, 1960.

1929

Sur la correspondance entre les frontières dans la représentation conforme. *Mat. Sb.* Vol. 36, No. 2, pp. 112-115. With Russian summary.

Sur un problème de M.P. Montel. *C.R. Acad. Sci., Paris* Vol. 188, pp. 689-691.

1930

Sur une méthode géometrique dans la représentation conforme. *Atti del Congresso internazionale dei matematici. Bologna 3-10 settembre 1928. Comunicazioni sezione I (C-D) - VI,* pp. 241-242. Zanichelli, Bologna.

Sur un problème de maximum dans la représentation conforme. *C.R. Acad. Sci., Paris* Vol. 191, pp. 827-829.

Sur la représentation conforme. *Ibid.* pp. 1426-1427. (In collaboration with V. Chepeleff).

Sur l'existence de la dérivée-limite. *Bull. Soc. Math. Fr.* Vol. 58, pp. 175-198. (In collaboration with P. Bessonoff).

1931

K voprosu o raschete neravnomerno nagruzhennogo, mnogopoletnogo lonzherona postoyannogo secheniia (On the question of the computation of a non-uniformly loaded multiplane longeron of constant cross-section). *Tekh. Vozd. Flota* No. 1, pp. 33-39.

Sur l'existence des dérivées-limites. *Mat. Sb.* Vol. 38, Nos. 3-4, pp. 51-58. With summary in Russian. (In collaboration with V. Holtzmann).

1932

O postroyenii potoka, obtekaiushchego dugu zadannoi formy (On the Construction of a Flow over an Arc of Given Form). Gos. Aviats. i Avtorakt. Izd., Moscow. 53 pp. with figs. (*Tr. TSAGI* No. 118).

1934

Ob odnoi ekstremal'noi zadache v teorii kryla aeroplana (On a Certain Extremum Problem in the Theory of an Airplane Wing). GTTI, Moscow-Leningrad. 40 pp. with figs. With summary in German. (*Tr. TSAGI* No. 155).

Programma kursa "Variatsionnoye ischislenie" (Program of the course "Variational computation"). Mekh.-Math. fac. MGU, Moscow. 2 pp. Litogr. Izd.

K teorii konforminykh otobrazhenii (On the theory of conformal mappings). *Tr. Fiz.-Math. Inst., Otd. Mat.* Vol. 5, pp. 159-245.

Sur deux questions extrémales. *Mat. Sb.* Vol. 41, No. 1, pp. 157-165. With summary in Russian.

Sur la représentation conforme. *Uch. zap. MGU* No. 2, pp. 39-41. With summary in Russian.

1935

Osnovy variatsionnogo ischisleniia (Fundamentals of Variational Calculation), Vol. I, Part 1, *Funktsii mnogikh peremennykh. (Functions of Many Variables)*. ONTI, Gl. Red. Obshchetekhn. Lit., Moscow-Leningrad. 148 pp. with figs. (In collaboration with L. A. Liustrenik).

Ibid. Vol. I, Part 2. ONTI, Gl. Red. Obshchetekhn. Lit., Moscow-Leningrad. 400 pp. with figs. (In collaboration with L. A. Liustrenik).

K teorii biplannoi korobki kryl'ev (On the Theory of a Biplane Box Wing). TSAGI, Moscow. 38 pp. with diags. (In collaboration with Ia. I. Sekerzh-Zenkovich and V. M. Shepelev). (*Tr. TSAGI* No. 153).

Geometricheskie voprosy teorii funktsii compleksnogo peremennogo (Geometrical questions in the theory of functions of a complex variable). In *Trudy II vsesoiuznogo matematicheskogo s''ezda. (Trans. of the Second All-Soviet Math. Congress)*, Vol. I, *Plenarnye zasedaniia i obzornye doklady (Plenary Session and Survey Reports)*, pp. 258-270. Akad. Nauk SSSR, Moscow-Leningrad.

Obshchaia zadacha o zhestkom udare o vodu (The general problem of a hard impact on water). In *Sbornik statei po voprosam udara o poverkhnost' vody (Collection of articles on questions of impact on the surface of water)*, pp. 5-12 with figs. ONTI, Gl. Red. Aviats. Lit., Moscow-Leningrad. (In collaboration with M. V. Keldish). (*Tr. TSAGI* No. 152).

Obzor rabot TSAGI po udaru tel o vodu. Teoreticheskie raboty (Review of TSAGI paper on the impact of bodies on water. Theoretical papers). In *Trudy I vsesoiuznoi konferensitsii po gidrodinamike (Trans. of the First All-Soviet Conference on Hydrodynamics)*, pp. 13-14. TSAGI, Moscow.

K teorii kryla aeroplana (On the theory of an airplane wing). In *Sbornik obshcheteoreticheskoi gruppy TSAGI (Collected Papers of the General Theoretical Group of TSAGI)*, pp. 37-38. TSAGI, Moscow. (*Tekhnicheskie zametki TSAGI* No. 45). (*TSAGI Technical Note* No. 45).

K teorii biplannoi korobki (On the theory of a biplane box). *Ibid.* pp. 39-40. (In collaboration with Ia. I. Sekerzh-Zen'kovich and V. M. Shepelev).

K teorii kolebliushchegosia kryla (On the theory of an oscillating wing). *Ibid.* p. 48. (In collaboration with M.V. Keldish).

K teorii biplannoi korobki kryl'ev (On the theory of a biplane box wing). In *Trudy III Vsesoiuznoi konferentsii po aerodinamike (Trans. of the Third All-Soviet Conference on Aerodynamics)*, 23-27 Dec. 1933. Part 2, pp. 202-203. TSAGI, Moscow. (In collaboration with Ia.I. Sekherzh-Zen'kovich and V.M. Shepelev).

K teorii kolebliushchegosia kryla (On the theory of an oscillating wing). *Ibid.* p. 223. (In collaboration with M.V. Keldish).

O nekotorykh svoistvakh odnolistnykh funktsii (On certain properties of univalent functions). *Dokl. Akad. Nauk SSSR* Vol. I, No. 1, pp. 1-2.

Sur quelques propriétés des fonctions univalentes. *Ibid.* pp. 2-4.

K teorii konformnykh otobrazhennii (On the theory of conformal mapping). *Dokl. Akad. Nauk SSSR* Vol. I, Nos. 2-3, pp. 85-87. (In collaboration with M.V. Keldish).

Sur la représentation conforme. *Ibid.* pp. 87-88.

Ob absoliutnykh konstantakh tipa A. Blokha (On the absolute constants of A. Bloch). *Dokl. Akad. Nauk SSSR* Vol. I, No. 5, pp. 279-282. (In collaboration with A. Bermant).

Sur les constantes absolues analogues à la constante de M. A. Bloch. *Ibid.* pp. 282-284.

Sur une classe de représentations continues. *Mat. Sb.* Vol. 42, No. 4, pp. 407-424, with figs. Summary in Russian.

Sur l'ensemble des valeurs d'une fonction analytique. *Ibid.* pp. 435-450. Summary in Russian. (In collaboration with A.F. Bermant).

O nekotorykh prilozheniiakh konformnykh otobrazhenii k gidrodinamike (On some applications of conformal mapping to hydrodynamics). *Tr. VVA RKKA* sb. 13, pp. 18-27.

Sur une classe de représentations continues. *C.R. Acad. Sci., Paris* Vol. 200, pp. 1010-1012.

1936

Sur les fonctions d'une variable complexe représentables par des séries de polynômes. Hermann, Paris. 64 pp. (Actualités scientifiques et industrielles, 441. La théorie des fonctions, V).

O semeistvakh odnolistnykh funktskii (On families of univalent functions). In *Trudy II Vsesoiuznogo matematicheskogo s''ezda (Trans. of the Second All-Soviet Mathematical Congress)*, Vol. 2, Sectional Reports, pp. 170-172. Akad. Nauk SSSR, Moscow-Leningrad.

O konstantakh Blocha (On Bloch's constants). *Ibid.* pp. 172-173. (In collaboration with A.F. Bermant).

O nepreryvnosti odnolistnykh funktsii v samknutykh oblastiakh (On the continuity of univalent functions in closed regions). *Dokl. Akad. Nauk SSSR* Vol. 4, No. 5, pp. 207-209. Literatura 4 nazv.

Sur la continuité des fonctions univalentes. *C.R. Acad. Sci. URSS* Vol. 4, No. 5, pp. 215-217. Littérature 4 noms.

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Sur les suites de polynômes harmoniques. *C.R. Acad. Sci., Paris* Vol. 202, p. 1149. (In collaboration with M.V. Keldish).

1937

O dvizhenii kryla pod poverkhnost'iu tiazheloi zhidkosti (On the motion of a wing below the surface of a liquid). In *Trudy Konferentsii po teorii volnovogo soprotivleniia (Trans. Conf. on the Theory of Wave Resistance)*, pp. 31-64, with figs. TSAGI, Moscow. Summary in English. (In collaboration with M.V. Keldish).

Ob ustoichivosti reshenii zadachi Dirikhle (On the stability of solutions of Dirichlet's problem). *Izv. Akad. Nauk SSSR, OMEN, Seriia mat.* Vol. I, No. 4, pp. 551-595. Summary in French. (In collaboration with M.V. Keldish).

O edinstvennosti zadachi Neimana (On the uniqueness of Neumann's problem). *Dokl. Akad. Nauk SSSR* Vol. 16, No. 3, pp. 151-152. (In collaboration with M.V. Keldish).

Sur l'unicité de la solution du problème de Neumann. *C.R. Acad. Sci. URSS* Vol. 16, No. 3, pp. 141-142.

O nekotorykh svoistvakh odnolistnykh funktsii (On certain properties of univalent functions). *Mat. Sb.* Vol. 2, pp. 319-326, with figs. Summary in French. (In collaboration with V.M. Shepelev).

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K teorii strui (On the theory of jets). *Dokl. Akad. Nauk SSSR* Vol. 18, Nos. 4-5, pp. 225-226.

Sur la théorie des sillages. *C.R. Acad. Sci. URSS* Vol. 18, Nos. 4-5, p. 225.

O nekotorykh svoistvakh struinykh techenii (On certain properties of jet flows). *Dokl. Akad. Nauk SSSR* Vol. 20, No. 4, pp. 235-237.

Sur quelques proprietes des courants discontinus d'un fluide. *C.R. Acad. URSS* Vol. 20, No. 4, pp. 235-237.

K teorii struinykh techenii (On the theory of jet flows). *Dokl. Akad. Nauk SSSR* Vol. 20, No. 4, pp. 239-240.

Sur la théorie des sillages. *C.R. Acad. Sci. URSS* Vol. 20, No. 4, pp. 239-240.

Ob odnom differentsial'nom priznake gomeomorfnykh otobrazhenii trekhmernikh oblastei (On a differential criterion of homeomorphic transformations of three-dimensional spaces). *Dokl. Akad. Nauk SSSR* Vol. 20, No. 4, pp. 241-242.

Sur une classe de transformations quasi-conformes et sur les sillages gazeux. *C.R. Acad. Sci. URSS* Vol. 20, No. 5, pp. 343-345.

O nekotorykh svoistvakh odnolistnykh funktsii s prilozheniiami k teorii strui (On certain properties of univalent functions, with applications to the theory of jets). *Mat. Sb.* Vol. 4, No. 3, pp. 391-458, with figs. Summary in French.

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